Guaranteed Trajectory Tracking using Flatness

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Introduction

In this talk we are interested in tracking the trajectory of differential systems when the system is time continuous. Its implementation can be greatly simplified when the system checks the flatness property. The proposed method then makes use of non-linear model-based predictive control [1] (a control-law strategy to direct the state of a cyber-physical system along a given trajectory predict). BoxRRT [6] is an algorithm based on RRT (Rapidly exploring Random Tree) Algorithm combined with interval analysis tools (e.g., guaranteed numerical integration). It computes an outer approximation of the states at each time interval k. It takes into account the model of the studied system and a map of static obstacles.

NMPC Among the control methods capable of tracking a reference trajectory, Nonlinear Model Predictive Control (NMPC) is well-adapted, especially in the presence of constraints on state and/or input variables [1]. The aim of NMPC is to determine a sequence of controls by solving a constrained optimization problem at time k over a prediction horizon n_p.

Differential flatness The idea of differential flatness was first introduced by Fliess et al. in 1995 [2]. A system is differentially flat if there exists a set of independent variables (equal in number to the dimension of inputs) referred to as the flat output such that all states and inputs of the system can be expressed in terms of this flat output and a finite number of its successive time derivative (resp. advances) for continuous-time (resp. discrete-time, see [4]) systems.

We consider the class of non-linear continuous systems described by:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \qquad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ $y \in \mathbb{R}^p$ are respectively the state, the input and the output of the system. The functions $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ and $h : \mathbb{R}^n \mapsto \mathbb{R}^p$ are nonlinear vector functions.

The system is said differentially flat if there exist a particular output (named flat output) $z \in \mathbb{R}^m$ which is a projection on m coordinates of the state x such that the state and the output of the system can be described using z and a particular number of its successive time derivatives:

$$\begin{aligned} z &= \varphi(x, u, \dot{u}, ..., u^m) \\ x &= \varphi_0(z, \dot{z}, ..., z^{(r)}) \\ u &= \varphi_1(z, \dot{z}, ..., z^{(r+1)}) \\ \dot{\varphi} &= f(\varphi_0, \varphi_1) \end{aligned} \qquad (2)$$

with r corresponding to the relative degree of the system. Then, the knowledge of the value of z and its successive time derivatives over time allows to characterize the state and the output of the system.

In [5], the computation of a guaranteed inner approximation of the set of the admissible controls using the flatness and NMPC has been addressed in the case of discrete time systems. The goal of the presentation is to extend this result in the case of continuous time

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system.

The method

Thanks to boxRRT, we are able to get a set of states values for which the system is guaranteed to reach a defined goal without colliding with static obstacles. When the system is flat, this result can be used to produce a set of values for the flat output and its time derivative and then allow to provide a set of admissible value for the input of the system. Indeed, if we consider $u(t)$ as piece-wise constant, we have

$$\begin{aligned} [\dot{x}_k] &= f([x_k], u_k) \\ [\ddot{x}_k] &= \frac{\partial f}{\partial x}([x_k], u_k).[\dot{x}_k] \\ &\vdots \end{aligned}$$

Using Eq. (2), we are then able to produce a set of controls that can be applied at time k. A particular control is then applied to the system (the midpoint of the characterized input set for instance).

Experiment

The method is illustrated using the Dubin's car model (see [3]) which is given by:

$$\begin{aligned} \dot{x} &= u\cos\theta \\ \dot{y} &= u\sin\theta \\ \dot{\theta} &= v \end{aligned} \quad (3)$$

In [3], it has already been shown that this system is flat with the flat output $z = (x, y)$ and

$$u = \sqrt{\dot{x}^2 + \dot{y}^2} \quad (4)$$

$$v = \frac{\ddot{y}\dot{x} - \ddot{x}\dot{y}}{\dot{x}^2 + \dot{y}^2} \quad (5)$$

All the system variables x, y, θ, u, v are thus expressed as function of $x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}$. Using boxRRT, we are able to provide a set of trajectories avoiding any obstacles and reaching a given goal (see Figure 1). This result is then used to define a control synthesis using NMPC.

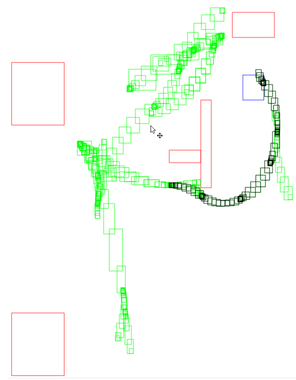

Figure 1: Example of result (in the phase space) of the boxRRT algorithm (red: obstacles; blue: goal; green: candidates paths; black: solution).

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