Prospects on the application of necessary optimality conditions on the resolution of the Goddard problem with unknown bounded parameters using interval arithmetics

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Introduction

Optimal control of aerospace systems is performed by modelling the considered system by dynamics depending on multiple uncertain parameters (for example, aerodynamic coefficients and maximal thrust). Usually, the optimal control problem is solved for the nominal values of these parameters and the robustness of the solution is demonstrated by dispersing the parameters around nominal values with Monte Carlo simulations. In addition to parameter uncertainties, the problem-solving method often introduces numerical approximation (for example the numerical solver of the ordinary differential equation representing the dynamics of the system or the optimization algorithm solving the optimal control problem).

Interval Arithmetics has shown its ability to address several control problems, providing validated solutions while dealing with method uncertainties (numerical approximations) as well as with model uncertainties (unknown parameters). The Pontryagin Maximum Principle [3] (PMP) provides necessary optimality conditions for the resolution of optimal control problems by transforming an optimal control problem into a zero-finding problem : the dynamics of the system is extended with a co-state and necessary conditions are given by the PMP on that co-state, and the initial value of the co-state vector is the unknown to be found by the zero-finding algorithm. This method has proven its efficiency and its precision compared to direct methods [3], but its convergence depends strongly on its initialization and a prior knowledge of the solution structure is needed.

Our goal is to address the return version of the Goddard problem, which consists in performing the landing of the first stage of a rocket while minimizing its fuel consumption, combining interval arithmetics and the necessary optimality conditions given by the application of the PMP. Although this goal has not been reached yet, this paper presents preliminaries results on simplified problems, exposes the challenges encountered and suggests further developments. The optimal reentry trajectory for the Goddard problem is presented in Figure 1 with the ballistic phases in blue, and in red the first boost for the inversion of the speed vector, the intermediate boost for the dynamic pressure constraint and the landing boost. The evolution of the dispersions along the complete trajectory (launch and landing) of a rocket are described in [1].

As a first step, a very simplified version of the Goddard problem is studied, namely a double integrator where $[k] = [\underline{k}, \overline{k}]$ is an interval parameter and u is the control in-

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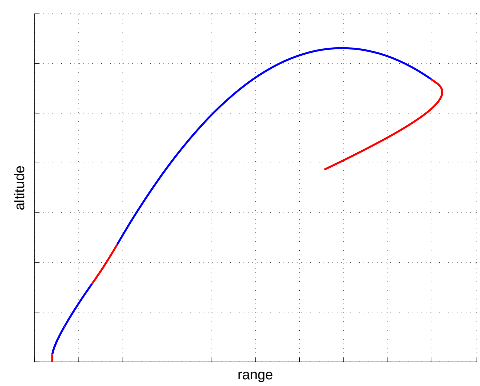

Figure 1: Optimal trajectory for the re-entry Goddard problem

put. This double integrator fits in with a Goddard problem without gravity and aerodynamic forces, and with a constant mass. The interval parameter $[k]$ represents the uncertainty on the maximal thrust force. Hence, the optimal control problem is

$$\min \int_0^T |u|dt, \begin{cases} \dot{r} = v, \dot{v} = [k]u, \\ r(0) = 0, v(0) = 0, \\ r(T) = r_T, v(T) \text{ is free}, \\ T \text{ is fixed}. \end{cases}$$

Two ways of combining interval arithmetics with the PMP are investigated : an *open loop* approach providing an enclosure on the system trajectory which can be used to assess robustness (regarding the interval parameter $[k]$), and a *closed loop* approach providing a closer enclosure of the optimal trajectories and can be used to initialize a non-interval algorithm.

Once the double integrator optimal control problem is solved, multiple approaches such as validated continuation methods are considered to solve the Goddard problem using this simplified version.

Open-loop approach

In this approach, the goal is to find the smallest initial co-state interval that contains every admissible co-state by combining numerical integration tools such as DynIbex[1] [2] and an algorithm to solve the zero-finding problem. Many algorithms are considered, for example Krawczyk method, forward-backward operators and branch algorithms. The enclosure of the solution of the zero-finding problem provides a validated initialization for the co-state vector. Issues like discontinuous control input, control saturation, pure state constraints and mixed constraints are to be studied in order to solve a practical optimal control problem like the Goddard problem.

Closed-loop approach

In this approach, dynamic programming is used to find a finer enclosure of the optimal trajectories : if the system measures its interval state vector at a certain time, a new optimal control problem is solved from this interval, providing a better enclosure of the solution. Due to its algorithmic cost, this approach is irrelevant for solving practical cases online, but it can improve the enclosure of the solution when it is applied offline and therefore provides useful information for the online guidance algorithm.

References

[1] L. Blackmore. Autonomous precision landing of space rockets. *The Bridge*, 46:15–20, Jan 2016.

[2] J. A. dit Sandretto and A. Chapoutot. Validated explicit and implicit Runge–Kutta methods. *Reliable Computing*, 22(1):79–103, Jul 2016.

[3] E. Trélat. Optimal control and applications to aerospace: some results and challenges. *Journal of Optimization Theory and Applications*, 154(3):713–758, 2012.

[1] http://perso.ensta-paristech.fr/~chapoutot/dynibex/