A Polytopic Box Particle Filter for state estimation of Non Linear Discrete-Time Systems

Thomas Gatto*[1], Luc Meyer[1], and Hélène Piet-Lahanier[1]

[1]Département Traitement de l'Information et Systèmes, ONERA, Université Paris Saclay

Keywords: Particle Filter, Intervals, Orthotopes, Ellipsoids, Estimation, Bounded noise, Set-Membership uncertainty, Uncertain dynamic systems

Introduction

State estimation of dynamic systems is commonly addressed by modelling the uncertainty as a stochastic variable, usually assumed Gaussian. For linear or non-linear systems, such problems are solved by using a classical (KF), an extended (EKF) or an unscented (UKF) Kalman Filter. For non-linear systems, particle filters have been developed to tackle non-Gaussian noise distributions. However, stochastic representation of errors is not immune to criticism as the probability density function is seldom known a priori. In set-membership estimation, process and measurement uncertainties are only assumed to vary within known bounds which makes this type of approach very robust to lack of probabilistic information. Various set structures have been used to characterize the variation domain of the system states, given the model structure and bounds. However, this results often in a pessimistic estimation, especially for multi-modal distributions. A more recent alternative method, first introduced by [1] consists in combining the versatility of the particle representation with the robustness of set-membership method. This translates in replacing the point particle by a box which results in reducing significantly the number of particles and the adverse effects of non-linearity. Box Particle Filter (BPF) estimators have already been applied in Simultaneous Localization and Mapping (SLAM) or mobile localization [1, 7]. However, the BPF provides a rather pessimistic solution due to the fact that the intervals have to be aligned along the state axis which result in loosing potential dependencies between the resulting estimate components. To address this issue, an improvement of the box description could be to combine this description with a more precise set characterization using either ellipsoidal [2, 4, 5] or polyhedral boundaries [6]. The aim of the present work is to build a new box particle filter based partially on polytopic description.

Problem Statement

Consider the following non linear discrete-time system:

$$\begin{cases} x_{k+1} &= f(x_k) + w_k \\ y_k &= h(x_k) + v_k \end{cases}, \qquad (1)$$

where $x_k \in \mathbb{R}^{n_x}$ is the state vector, $y_k \in \mathbb{R}^{n_y}$ the measurement vector, $f : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ a non-linear function and w_k, a process noise vector. We denote by n_x, n_w, respectively, the dimensions of the state and process noise vectors. The function $h : \mathbb{R}^{n_x} \to \mathbb{R}^{n_y}$ is a non-linear function and v_k a measurement noise vector. Dimensions of the measurement and measurement noise vectors are respectively n_y and n_v.

Assumption 1. The disturbance terms w_k and v_k are assumed to be unknown but bounded (UBB) noises:

$$|w_{k,i}| \leqslant \varepsilon^w_{k,i}, i = 1, \dots, n_w \iff \|w_k\|_\infty^{\varepsilon_k^w} \leq 1, \qquad (2)$$

*Corresponding author.

$$|v_{k,i}| \leqslant \varepsilon^v_{k,i}, i = 1, \ldots, n_v \Longleftrightarrow \|v_k\|_\infty^{\varepsilon^v_k} \leq 1. \quad (3)$$

Definition 1. A real interval, denoted $[x]$, is defined as a closed and connected subset of $\mathbb{R}$ and a box $[X]$ of $\mathbb{R}^{n_x}$ as a Cartesian product of n_x intervals: $[X] = [x_1] \times [x_2] \times \ldots \times [x_{n_x}] = \times_{i=1}^{n_x} [x_i]$.

Definition 2. An n-dimensional polyhedron P is defined as a set of n_p vertices $\mathbb{V}_i, i = 1, \ldots, n_p$ and n_h supporting hyper-plans $\mathbb{H}_j$.

Each of the n_h hyper-plans is defined by $\{x \in \mathbb{R}^n | a_i x = b_i\}$, where $a_i^T \in \mathbb{R}^n$ and $b_i \in \mathbb{R}$ Therefore, a n-dimensional polyhedron P supporting n_h hyper-plans is defined by :

$$\{x \in \mathbb{R}^n | Ax \leq b\}, \quad (4)$$

where $A \in \mathbb{R}^{n_h \times n}$, a_i the i-th row of A, $b \in \mathbb{R}^{n_h}$ and b_i the i-th component of b.

Proposed algorithm

The algorithm is based on the BPF algorithm. The main originality consists in modifying the update step of the BPF by replacing the measurement boxes by polytopes to improve accuracy.

Initialization

As in the BPF, the initialization consists in creating N_p box particles from the initial box with minimum intersection and equivalent weights.

Prediction

In this step, each state predicted particle is computed based on the previous state estimated particle, via a classical interval propagation.

Measurement update

The observation function h is linearized at the center $\hat{x}_k$ of the predicted box:

$$h(x_k) = h(\hat{x}_k) + C_k(x_k - \hat{x}_k) + o_k, \quad (5)$$

where $C_k = \frac{\partial h(\hat{x}_k)}{\partial x}$; o_k is the linearization error. The measurement bounds $[m_k]$ are obtained as $[o_k] + [v_k]$. For each measurement y_k, two bounding hyperplans are defined as $h(\hat{x}_k) + C_k(x_k - \hat{x}_k) = y_k + min([m_k])$ and $h(\hat{x}_k) + C_k(x_k - \hat{x}_k) = y_k + max([m_k])$.

Using the approach described in [6], the measurement update step consists in computing the feasible set for each particle by intersecting the predicted box particles with the two half spaces associated with each of the bounding hyperplans. The volumes of the resulting polytopes are computed as in [3], and will be used as weight for each polyhedron particle.

Estimation

At the k-th step, the state is usually approximated using the weighted particles, as $\widehat{x}_k = \sum_{i=1}^{N_p} w_k^i x_k^i$. In the case of box particles, so on the BPF, the state is actually computed as $\widehat{x}_k = \sum_{i=1}^{N_p} w_k^i C_k^i$, where C_k^i is the center of the box particle i. However, in our proposed filter, the new estimated state is computed as the center of the polytope i which is obtained as $C_k^i = \frac{1}{n_p} \sum_{j=1}^{n_p} \mathbb{V}_{k,j}$ where $\mathbb{V}_{k,j}$ is the j-th vertice of the polytope i at time k.
Similarly to the BPF, the associated covariance matrix is given by $\widehat{P}_k = \sum_{i=1}^{N_p} w_k^i (\widehat{x}_k - x_k^i)(\widehat{x}_k - x_k^i)^T$.

Resampling

The resampling phase consists in eliminating polytopes associated with the lowest weights, and in dividing the polytopes associated with the highest weights. These weights are obtained by computing the volume of each polytope. After selection of the polytopes to be kept, each of those is approximated by the smallest box containing it. Figure 1 illustrates the measurement update phase. It can be seen that the polyhedral update (in green) makes the resulting estimation uncertainty less pessimistic than with the classical Box Resampling (in black).

Several examples of non linear model estimation have been tested to evaluate the average precision improvement resulting from the use of the new method.

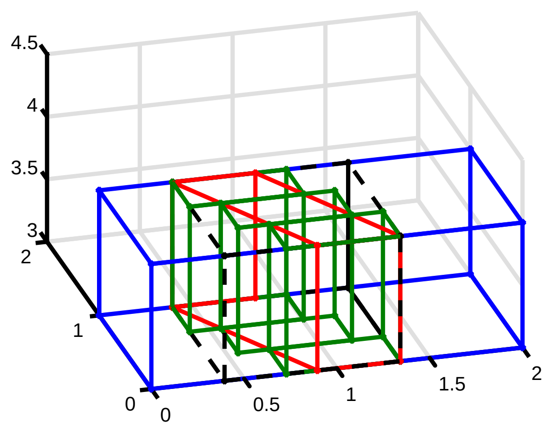

Figure 1: Illustration of the measurement update using polyhedrons. Blue: predicted box. Red: half spaces associated with each of the bounding hyperplans. Green: set of new boxes after resampling. Black: set that would be obtained with classical Box Particle Filter.

Conclusion

In this paper, improvement of box particle filter based on polytopic measurement updating is proposed. Different examples of application have been compared with the BPF and the results are promising. The estimate is more precise, especially if all the variables are measured. However, for now, the computing time is still uncertain because it depends on the dimensions of the state and the measure. Future work includes analysis of the computation of the bounds on measurements allowing the best compromise between reliability and precision. Evaluation of weights depending not only on the volume of the resulting polytopes is also under study.

References

[1] F. Abdallah, A. Gning, and P. Bonnifait. Box particle filtering for nonlinear state estimation using interval analysis. *Automatica*, 44(3):807–815, 2008.

[2] Z. Bo, Q. Kun, M. Xu-Dong, and D. Xian-Zhong. A new nonlinear set membership filter based on guaranteed bounding ellipsoid algorithm. *Acta Automatica Sinica*, 39(2):146–154, 2013.

[3] J. B. Lasserre. An analytical expression and an algorithm for the volume of a convex polyhedron inr n. *Journal of optimization theory and applications*, 39(3):363–377, 1983.

[4] B. T. Polyak, S. A. Nazin, C. Durieu, and E. Walter. Ellipsoidal parameter or state estimation under model uncertainty. *Automatica*, 40(7):1171–1179, 2004.

[5] E. Scholte and M. E. Campbell. A nonlinear set-membership filter for on-line applications. *International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal*, 13(15):1337–1358, 2003.

[6] E. Walter and H. Piet-Lahanier. Exact recursive polyhedral description of the feasible parameter set for bounded-error models. *IEEE Transactions on Automatic Control*, 34(8):911–915, 1989.

[7] P. Wang, P. Xu, P. Bonnifait, and J. Jiang. Box particle filtering for slam with bounded errors. In *2018 15th International Conference on Control, Automation, Robotics and Vision (ICARCV)*, pages 1032–1038. IEEE, 2018.